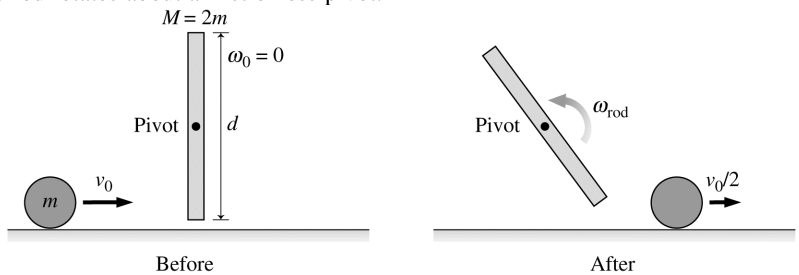
13.83. Model: For the (billiard ball + rod) system the angular momentum is conserved. The ball rolls without slipping and the rod rotates about a frictionless pivot.

Visualize:



Solve: **(a)** The initial angular momentum about the pivot due to the ball moving at $r = \frac{1}{2}d$ is $L_i = mv_0(\frac{1}{2}d)$ and the final angular momentum about the pivot is

$$L_f = I_{rod}\omega_{rod} + m(v_0/2)(d/2) = \left(\frac{1}{12}\right)(2m)(d^2)\omega_{rod} + mv_0\frac{d}{4}$$

Using $L_f = L_i$, we have

$$\frac{1}{12}(2m)(d^2)\omega_{rod} + mv_0\frac{d}{4} = mv_0\frac{d}{2} \Rightarrow \omega_{rod} = \frac{\left(\frac{1}{4}mv_0 d\right)}{\left(\frac{1}{6}md^2\right)} = \frac{3v_0}{2d}$$

(b) The initial kinetic energy $K_i = \frac{1}{2}mv_0^2$ and the final kinetic energy is

$$K_f = \frac{1}{2}m(v_0/2)^2 + \frac{1}{2}I_{rod}\omega_{rod}^2 = \frac{1}{8}mv_0^2 + \frac{1}{2}\left(\frac{1}{12}(2m)d^2\right)\left(\frac{3v_0}{2d}\right)^2 = \frac{1}{8}mv_0^2 + \frac{3}{16}mv_0^2 = \frac{5}{16}mv_0^2$$

Because $K_f \neq K_i$, the mechanical energy of the system is not conserved.